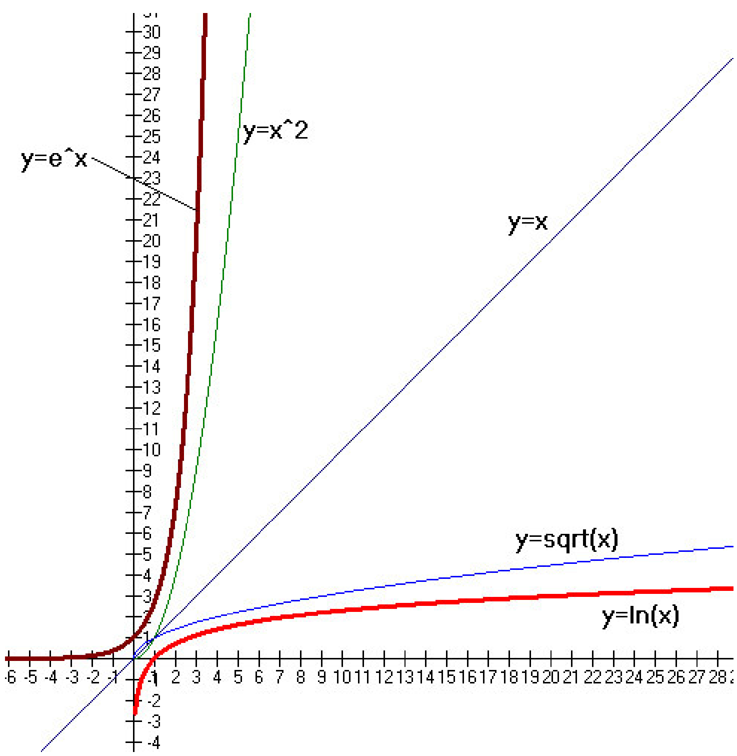
THIRD LECTURE: Flight Equations, Mass Ratio, Future Rockets

During our last adventure we introduced the concept that at any one instant in time, there are FOUR FORCES that act in concert on our rocket when it is in ATMOSPHERIC flight. I showed you two new forces—the AERODYNAMIC forces LIFT and DRAG, and we talked a lot about 'rates of change' and the SQUARE of a number.

Today what we're going to do is make some simplifying assumptions and see if we can come up with EQUATIONS that we can use to predict the flight path—trajectory—of our rocket.

Our Old Friend the Coordinate System—and Some New Friends to Help Us Understand More About 'Rates of Change'

I'm going to draw on the whiteboard, and you have here below a better copy, of the X-Y coordinate system from before when we discussed 'battleship.' I want to introduce you to the concept of a 'function' and several specific functions that are so important—not just to rocketry, but to all of science and nature.



I'm going to PLOT numbers… you can see the 'X' axis is horizontal, and the 'Y' axis is up and down, or vertical.

The diagonal line is what you get when ' y = x '.

The ' ∧ ' symbol is because on the internet or some computers the fonts I might use might get all mixed up, but what it means is the exponential function, 'e' is RAISED to the 'x' POWER (e^X).

[If you've already studied the negative number line—we won't discuss it today.]

Anyway, in this one little picture I can explain to you so many things! That's the power and beauty of MATHEMATICS. Unlike the written word, math says a lot with very little writing.

Remember when we talked about the force of DRAG and how it increases with the SQUARE of the velocity? Take a closer look at the graphs. Do you see the ' y = x^2 ' ? That's our SQUARE of the number function, and as you can see as X gets bigger, Y gets bigger MUCH FASTER. Compare the curve with the one for 'y = x' and you'll see what I mean.

If you like, make a table of the 'x,y' pairs so you can get a feel for what these graphs are showing you. Your table might look something like this below:

-------- VALUE OF 'y' WHEN 'x' is… (to only one decimal place)-----------

X	X = Y	Ln(x)	Sqrt(x)	x^2	$e^{(x)}$
1	1	0	1	1	2.7
2	2	0.7	1.4	4	7.4
3	3	1.1	1.7	9	20
4	4	1.3	2	16	54
…					

In general, whether we square a number, or raise it to some OTHER POWER (like the CUBE for instance, meaning raise the number to a power of '3' instead of '2') you can see that the of the curve looks pretty close to the shape of the curve for what we call the EXPONENTIAL, or 'e' –a really magic little number handed down to us from some very, very smart people who lived way back in the 16th and 17th Centuries (that's 400 years ago if you're keeping score!)

Now you might not recall, but in your SLIDE RULE NINJA course, I also introduced a concept familiar to some of you, called an INVERSE OPERATION. That's when you do the OPPOSITE of a math function. In particular, I told you that in math and on your slide rule, the inverse operation of MULTIPLICATION is DIVISION. Well, look at your graph. Notice how the shapes of the curves for the SQUARE ROOT and the 'Ln' or Logarithm functions are shaped like they were mirror images of the SQUARE and the 'e' functions?

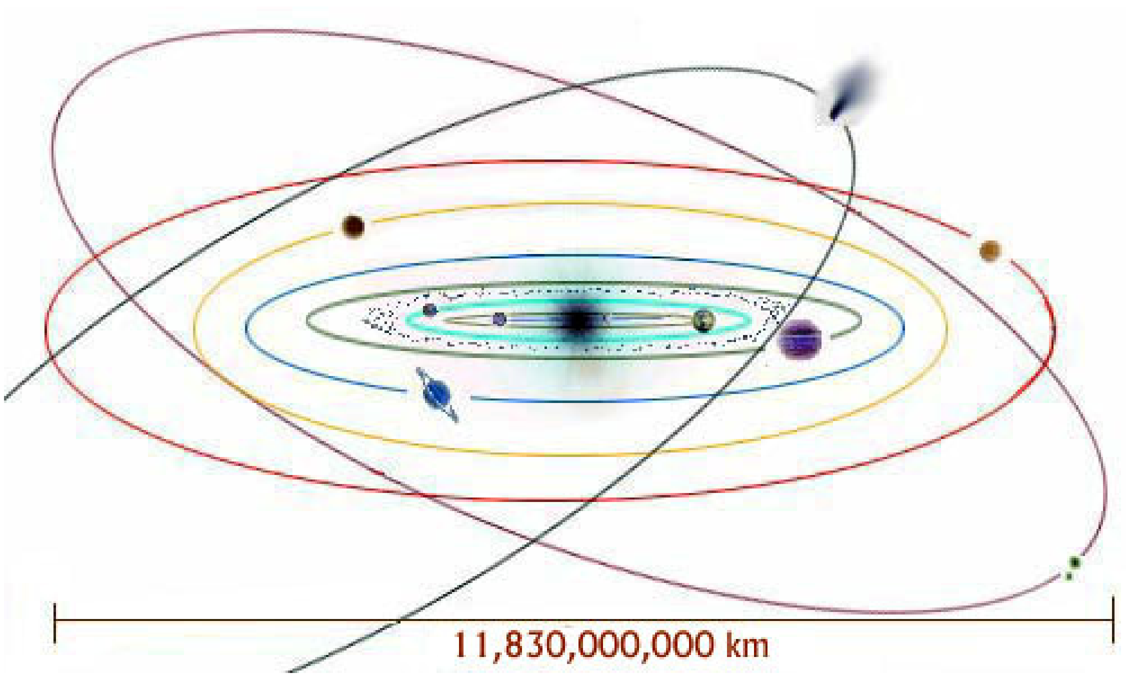
Well, it turns out that these INVERSE functions are extremely handy too—as we've already seen in the case of our slide rules and the power of LOGARITHMS –the INVERSE of the EXPONENTIAL function- to change multiplication and division problems into simple addition (and subtraction). What I want to show you briefly, is that for the SQUARE ROOT, or the LOG functions, as X gets larger, Y ONLY GETS LARGER BY the TINIEST LITTLE BIT.

Keep these important functions in mind—at least their graphs in mind—as we mention concepts later in today's discussion.

Space Voyagers –Will We Ever Arrive? About Distance, Time and Speed

Before we dive into more of the math, let's talk just a bit about space, and the incredible distances between heavenly bodies that we might want to go to. I want to give you a feeling for why the concepts you're learning about today are relevant to even what some of our most advanced NASA and European space scientists are doing right now.

Now here's a picture of our Solar System something like what I know you've all seen.

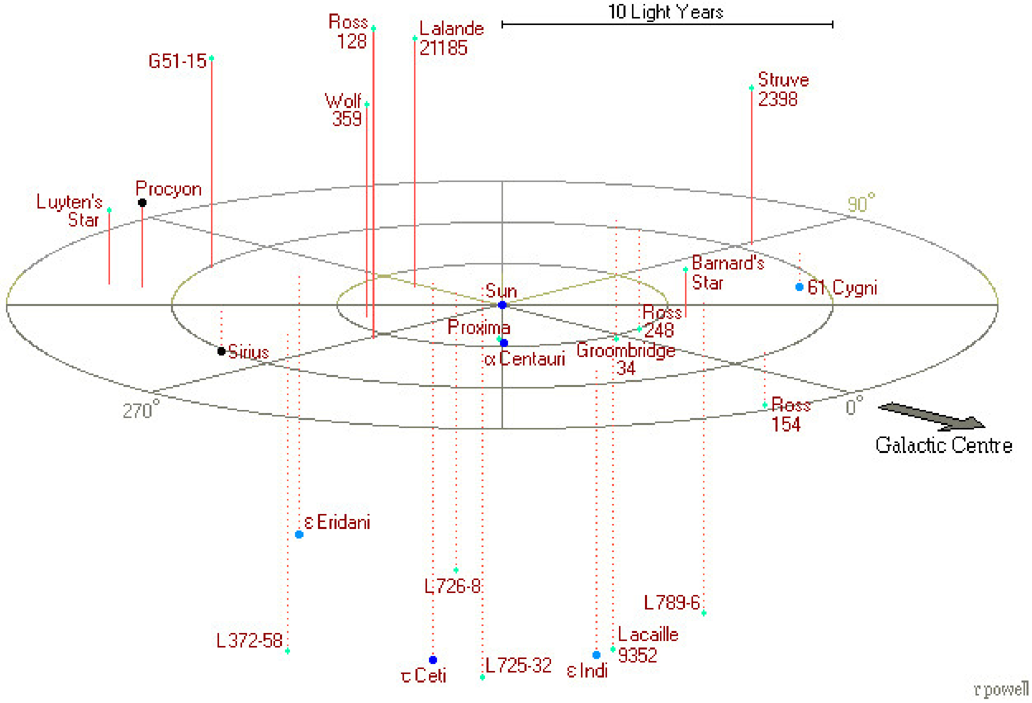


What I want to point out to you is that almost NOTHING about it is right—except at least, the artists got the planets in the right order, mostly. Now don't blame the artist—someone told him to put all the planets on the same page—even though if he drew them 'to scale' there is no way to do it!

SO… how should this look? If we were to draw our Solar System 'to scale' where the Sun was reduced to the size on this page, the EARTH WOULD BE 2 FEET AWAY! We would have to draw the Earth at a size of one-fifth the period at the end of this sentence! Jupiter, the largest planet by far, would be a mere dot, but it would be so far away from this page that you might want to take a bicycle to get to it.

What I'm trying to impress upon you is the following: While I like Star Trek and the SciFi channel a lot, it's just not realistic to think someone is going to the outer Solar System, let alone the next nearest star, anytime soon. That's until we solve the problem of how to travel very fast!

Now there are plenty of really interesting sites on the web that show examples of our Solar System to scale, so I won't repeat them here. Instead, here's a picture of the stars that are nearest to our Sun, all located within our Galaxy—the Milky Way.



Because these distances are so great, astronomers use a measurement called a "LIGHT YEAR" –that is, the distance one would travel if she were to go at the speed of light, which is approximately 300,000,000 meters per second TIMES the number of seconds in a YEAR!

The speed of light travelling in a vacuum (which space pretty much is) is called "c" which is ANOTHER example of one of those math quantities we call a CONSTANT (because it's value remains the same).

In fact, Einstein's "Special Theory Of Relativity" postulated for the first time that NOTHING can travel faster than light[1]. 'c' is the "ultimate" speed limit in the universe!

Now I said that no one will be travelling any time SOON to the edge of our Solar System, but the fact is mankind has already sent four spacecraft—launched way back in the 1970's when your parents and I still had flowers in our hair. Two of those spacecraft—Voyagers '1 & 2' have explored the outer planets, and are now at the edge of the 'heliosphere' –the region where our Sun's solar 'wind' pretty much hits interstellar space.

---

[1] If you read a lot, you might be tempted to think that some scientists believe that one can actually travel faster than 'c'. No. What you are reading is material that someone adapted from Einstein's "General Theory of Relativity" –which postulates that SPACE (distance) can be "folded" by extremely strong gravity sources, and that one therefore takes a "shortcut" to get somewhere, thereby appearing to travel faster than light.

These two spacecraft are still, after more than 25 years, taking readings and faithfully sending the data back to Earth! Although, it takes more than 90 minutes (at 'c' or the speed of light!) for the data to reach us.

The Ideal Rocket Equation and 'Mass Ratio'

Now today you'll (finally) be getting your rocket. Like me and just about everyone else I know who has had the pleasure of launching one, what most of us want to do is launch a rocket that goes as high up as we can make it go, travels fast and is exciting to watch. I know very few people who even cared (on their first launch) whether the darn thing came back or not! Fortunately for you, I promise yours will.

Now maybe some day, you'll find yourself wondering if you could be one of the people who will be sending spacecraft to Mars, the outer solar system, or heaven knows, maybe even be riding in one! You'll really want to know how you can make a rocket that will get you there, especially now that we know how FAR AWAY 'there' is. Oh, and of course, you might want to get BACK!

Rocket scientists have known about an equation that tells us what the 'speed limit' of our rocket will be, since the days of a guy whose name was Tsiolkovsky back at the turn of the last century. To no one's surprise, this equation is called the IDEAL ROCKET EQUATION.

It goes like this:

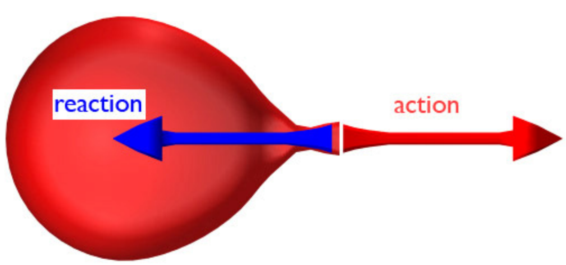
From Newton's SECOND and THIRD LAWS we'll set up an equation (an EQUALITY) in order to conserve energy.

(In class discussion note)

(mass of rocket) • (rocket velocity) = (mass of rocket propellant) • (exhaust velocity)

(REACTION) **equals** **(ACTION)**

You'll notice this equation is just the same idea as the picture of the balloon you've seen before…



And if you really think about it, this makes complete sense: we heated up a propellant until it turned into a gas, its mass was expelled with a high velocity, and the equal but opposite reaction is the mass of the ROCKET which moves off in the other direction, though because its mass is much greater, it doesn't go as fast as the hot gas!

Now, the "mass of rocket propellant" from the equation above is the same number as the mass of the 'full rocket' MINUS the mass of the 'empty rocket' –because the difference is the propellant that's been burned up. Next what I'll do is move the terms of our equation around and using calculus[2] to find the rate of change, I get the more common form of the IDEAL ROCKET EQUATION:

$$\Delta v = v_e \ln \frac{m_0}{m_1}$$

' Δ' means "the change". I read the equation this way: "delta V, or the change in the rocket's velocity, is equal to the velocity of the propellant TIMES the LOGARITHM of the **mass ratio**, or mass of the rocket 'full' divided by mass of the rocket 'empty'." Our friend the logarithm pops up because as you remember logs are a way of converting division into subtraction, and it works the other way around also: subtraction problems can become division problems!

This is called the IDEAL ROCKET EQUATION because right now, we're NOT considering either the WEIGHT FORCE or the DRAG FORCE—both of which operate in the opposite direction from where we want our rocket to go.

Now what this equation will tell us is, in effect, for the type of propellant (fuel plus oxidizer) we are using, the type of rocket engine and the amount of propellant our rocket can carry… WHAT IS THE ULTIMATE SPEED LIMIT of our ROCKET! This would apply if we were in the vacuum of space—where there are no aerodynamic forces, and the force of gravity between distant bodies is so small that we could afford to momentarily "ignore" it.

Puttling Space Voyagers and Mass Ratios Together: Future Spaceflight

Now I hope you remember back to our earlier lecture, when I talked about just how fast—how much Velocity—you have to have in order to make Earth Orbit. Or maybe you're thinking, boy, I want to be the one to go to Mars, or to discover what's under that "ocean" on Jupiter's moon Europa.

And I know you're thinking, "Gee—all I have to do is put a big enough rocket engine on this sucker and maybe I can get my rocket into orbit!" Well, I'm sorry to say, the rocket equation tells us it just isn't that easy. Let's see why.

[Please turn to the next page]

[2] Though beyond what we want to talk about in this lecture, Calculus—an advanced form of mathematics invented by our old friend Newton and another mathematician, Liebnitz—allows us to calculate the rates of change of things. If you have some exposure to this or want to learn more, ask me, and I'll be happy to point you in the right direction!

$$\Delta v = v_e \ln \frac{m_0}{m_1}$$

- Δv: Speed (velocity) we WANT to travel
- v_e: How 'advanced' is our propulsion system (velocity of exhaust)
- m_0: Full rocket mass
- m_1: Empty rocket mass

Go Faster: "more" ΔV

Ve must be "higher"…

model rocket ~2,000 m/s, shuttle solid ~2,500 m/s, shuttle liquid ~4,400 m/s, "Ion" ~29,000 m/s, "nuclear fusion" ~300,000 m/s

Ln goes up VERY SLOWLY (see the graph)… so…

m_0 must be HUGE (most of the rocket's mass must be propellant)
m_1 must be PUNY (less mass of payload, very strong but lightweight structures)

What ΔV to get to Low Earth Orbit (LEO) ? 4,500 m/s !!!

So as you can see, the name of the ballgame is "Mass Ratio" or the m_0/m_1 term.

Now , another way to describe the speed we want to go, is by expressing **ΔV** as a multiple of **Ve.** If we do this, I want to show you just how "big" our rocket has to be (how much propellant it needs to carry) to get to LEO:

Mass full / Mass empty	ΔV in multiples of **Ve**
10 times	2.3 times Ve
100 times	4.6 times Ve
1000 times	6.9 times Ve

So, to answer our question, to get our rocket to LEO, assuming there was NO GRAVITY and NO DRAG to overcome, we would need a “mass ratio” –that is, the weight of our rocket with propellant DIVIDED by the weight of our rocket after the propellant is all burned up—of about 10 TIMES.

EXERCISE:

Do the math on your slide rule, from the first line of the table above:

2.3 times Ve, assume Ve is a high-end model rocket engine with 2,500 m/s propellant…

which gives us about (2.3 times 2,500 m/s) is __________ m/s which is just above what we need for LEO (4,500 m/s).

Well, do you think you can get 100 times the mass of your model rocket engine into your rocket, and still have the darn thing hold together during flight?

So how in the world does NASA get the space shuttle up into space?

We’ll talk about this during our class exercise. For now, that ends this lecture!